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Comparison of Boundary Element and Finite Element Approaches to the EEG Forward Problem

J. Vorwerk[1], M. Clerc[2], M. Burger[3], C.H. Wolters[1]
j.vorwerk@uni-muenster.de

[1] Institute for Biomagnetism and Biosignalanalysis, University of Münster, 48149, Münster, Germany
[2] Inria Sophia-Antipolis—Méditerranée, 06904, Sophia-Antipolis Cedex, France
[3] Institute for Computational and Applied Mathematics, University of Münster, 48149, Münster, Germany

Abstract

The accurate simulation of the electric fields evoked by neural activity is crucial for solving the inverse problem of EEG.

Nowadays, boundary element methods (BEM) are frequently applied to achieve this goal, usually relying on the simplification of approximating the human head by three nested compartments with isotropic conductivities (skin, skull, brain). Here, including the highly-conducting cerebrospinal fluid (CSF) is a difficult task due to the complex geometrical structure of the CSF, demanding a high number of additional nodes for an accurate modeling and thus a strongly increased computational effort. Though, CSF conductivity is well-known and nearly not varying inter-individually and its significant influence on EEG forward simulation has been shown.

The CSF can be included at negligible computational costs when applying finite element (FE) forward approaches.

In this study we compare the accuracy and performance of state-of-the-art BE and FE approaches in both artificial and realistic three layer head models, showing that all approaches lead to high numerical accuracies. Furthermore, we give an impression of the significant influence of modeling the CSF compartment as disregarding this compartment leads to model errors that lie clearly above the observed numerical errors.

1 Introduction

The goal of EEG source reconstruction is to determine the active brain areas from measured potentials at the head surface [1–3]. To solve this problem, the accurate simulation of the electric field evoked by dipole sources in the brain is crucial. Therefore, it is necessary to apply numerical methods that are able to take the realistic shape of the head into account as several studies have shown [1], [2], [4–8].

One approach to achieve this goal are boundary element methods (BEM) [1], [4], [9], [10]. Approximating the head by a volume conductor consisting of compartments with constant, isotropic conductivities, the forward problem can be transformed to a boundary integral equation, which can then be solved numerically. A double layer BE approach was developed by Geselowitz [1]. To improve numerical accuracy, the isolated skull approach (ISA) has been proposed [4]. In our study we make use of the SimBio-toolbox[1], which implements a *linear collocation ISA BE approach* [2].

The recently proposed *symmetric BE approach* is based on a reformulation of the boundary integral equation using Newtonian potentials [9], [10]. It is implemented in the open-source toolbox OpenMEEG[2]. It has been shown that the symmetric BEM outperforms the double-layer BEM in three layer sphere models. Using an adaptive integration scheme, the achieved accuracy can be further increased [9], [10].

These approaches have the advantage of a relatively standardized model generation (three nested and non-intersecting compartments) and a relatively low computational complexity.

As a drawback of BE methods must be seen that anisotropy, e.g., in the white or gray matter compartment, whose influence on source analysis has been shown [8], cannot be taken into account. Furthermore, the distinction of tissue structures like, e.g., skull compacta and spongiosa [6] or the brain surface, which is needed to take the current-channeling CSF compartment into account [7], [11], [12], leads to an increase in computational complexity and memory demand due to the high mesh resolution needed to resolve their shape correctly. Additionally, most current BEM implementations are restricted to nested shell topographies.

The finite element methods developed for source analysis do not suffer from these specific problems, being able to handle both anisotropy and complex geometries [3], [5–7], [13], [14]. The computational complexity can be reduced by the introduction of transfer matrix or reciprocity approaches and fast iterative solver methods, which lead to an overall linear dependence on the number of FE nodes, enabling the practical usage of FE methods [3], [13], [14]. Three different FE approaches are considered here, differing in the treatment of the singularity introduced through the source term. Two of them are direct approaches, i.e., the source is approximated locally by a distribution of electrical monopoles. This distribution can be computed by means of an application of the law of St.

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[1] https://www.mrt.uni-jena.de/simbio/

[2] http://www-sop.inria.fr/athena/software/OpenMEEG/

Venant to electromagnetism (*Venant FE approach* [3], [5], [13]) or by applying partial integration (in a distributive sense) to the right hand side of the forward problem (*partial integration FE approach* [3], [13]). Under the assumption that the conductivity is constant in a neighborhood of the source, the forward problem can be reformulated. Using the analytical solution of the forward problem in an infinite homogeneous conductor, the source singularity can be subtracted so that only a correction potential remains to be calculated numerically, the so-called *subtraction FE approach* [3], [13], [14].

This paper has the goal to compare the approaches (1) double-layer (linear collocation ISA) BEM, (2) symmetric BEM, (3) Venant FEM, (4) partial integration FEM and (5) subtraction FEM with regard to accuracy and computational complexity in three layer sphere and realistic head model scenarios. An important further goal is to set the numerical errors in relation to the model errors due to neglecting the effect of the highly-conductive CSF.

2 Methods

Two different scenarios are used for our studies. As a first step, we carry out calculations in a three layer sphere model. Here, an analytical solution exists and can be used as reference. Additionally it enables us to demonstrate the influence of the highly conducting CSF by calculating the pure analytical error of neglecting this compartment. Afterwards, we run a similar comparison in a realistically shaped three layer head model.

2.1 Sphere Model

To evaluate the BE methods, we created triangulations of concentric spheres with radii of 80 mm, 86 mm and 92 mm representing the boundaries between the compartments brain, skull and skin. Each surface mesh consists of 2,056 nodes leading to an overall node count of 6,168. As conductivities we choose 0.33 S/m, 0.01 S/m and 0.43 S/m, respectively [6], [7]. For the CSF we assume 1.79 S/m [11].

We used TetGen [15] to create tetrahedral meshes for the FE methods. TetGen allows to impose a volume constraint, limiting the size of the tetrahedra, and a quality constraint to prevent badly shaped elements. Based on triangulations of the sphere surfaces, a constrained Delaunay tetrahedralization was performed. We created a mesh with a homogeneous resolution resulting in 818,048 nodes and 5,097,930 elements, which we used with the direct approaches. Here, our goal was to roughly adjust the resolution so that the computation times of the direct FE and the BE approaches are comparable.

For the subtraction approach we created a mesh with a high resolution in the outer two compartments and no volume constraint in the innermost compartment, ending up with 497,108 nodes and 3,027,991 elements [13], [14]. The smaller number of nodes was chosen due to the much higher computational effort of the subtraction FE approach compared to the direct FE approaches [13], [14].

Finally, 522 electrode positions were distributed regularly on the outermost surface and test sources were placed in 1 mm steps at distances between 2 mm and 77 mm to the origin of the spheres. At each distance we distributed 125 sources randomly, to allow for a statistical evaluation indicating the effect of mesh properties in the vicinity of the source and resulting error variability, which is especially existent for the direct FE approaches. Both radial and tangential source directions were computed. We will only depict and discuss the results for the tangential sources here, the results for radial sources are similar; for an extensive evaluation refer to [3].

We evaluate the numerical accuracy using the well-known error measure RDM (relative difference measure), i.e., the l_2 error between normalized numerical and reference solution, and the ln(MAG) (logarithmic magnitude error), i.e., the logarithm of the ratio between norm of numerical and reference solution, having the advantage of symmetry in contrast to the classical MAG [8].

2.2 Realistic Head Model

Furthermore, we created a realistically shaped three compartment head model on the basis of a T1-weighted MRI using CURRY[3]. We extracted surfaces with 2,219, 1,814 and 2,879 vertices for skin, outer skull and inner skull, respectively. While the surfaces could directly be used with the BE approaches, we again performed a tetrahedralization based on these surfaces using TetGen to create a tetrahedral model for the FE approaches. This resulted in 933,038 nodes and 5,891,852 elements for the direct approaches and 653,664 nodes and 4,075,056 elements for the subtraction approach. Since no analytical solution exists in this scenario, we also created a high resolution FE model to calculate a reference solution. This resulted in 2,242,186 nodes and 14,223,508 elements. Finally, we constructed a high resolution FE model where the CSF compartment was taken into account to evaluate the influence of modeling/not modeling the CSF (2,268,847 nodes, 14,353,897 elements).

18,893 test sources were distributed regularly on an extracted white/gray matter interface. This assures a sufficient distance between source positions and brain surface, in order to avoid numerical errors as a consequence of sources being too close to a conductivity jump when taking into account the CSF compartment. The surface normals were chosen as source directions. The simulated potentials were evaluated using a realistic 80 sensor electrode cap.

3 Results

The sphere study (Fig. 1) underlines the results of previous evaluations. The symmetric BEM clearly outperforms the double layer approach, which is showing the worst performance of all tested approaches, having RDMs of up to 0.16 at the highest eccentricity (out of range) and a

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[3] http://www.neuroscan.com

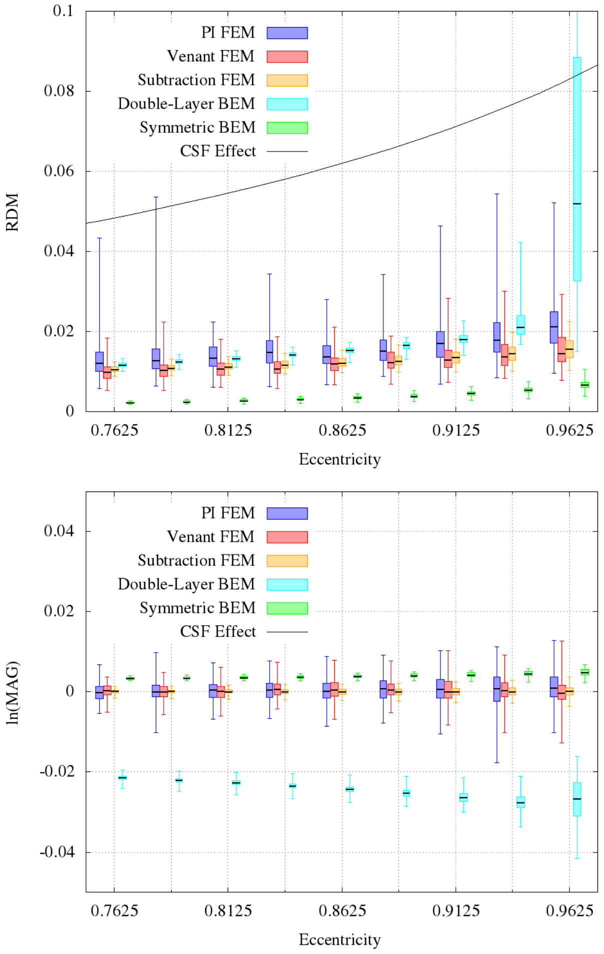

Figure 2: RDM and ln(MAG) for tangential dipoles in a three layer sphere model. CSF effect is out of range in the lower graph in positive y-direction.

ln(MAG) of under -0.02 for all eccentricities that is also varying strongly at high eccentricities. In contrast, the symmetric BEM shows the best result of all approaches, having a RDM below 0.01 and a ln(MAG) only slightly higher than for the FE approaches at a still very small level and with little variety.

Venant and subtraction approach perform nearly equally well, the RDM is constantly lower than 0.02 except for some outliers at high eccentricities. The median of the ln(MAG) is constantly around 0 at all eccentricities for both approaches. While the errors of the Venant approach generally show a higher variety, which can be interpreted as a consequence of the dependency on the local mesh structure, the subtraction approach achieves a small error range at every eccentricity in both modalities. As a consequence, the maximal RDM of the Venant approach is higher than those of the subtraction approach at every eccentricity, while the median of the errors is lower. The partial integration approach shows the worst results of the FE approaches, but is still performing reasonably well with RDM medians below 0.025 and maxima below 0.06. The median of the ln(MAG) is nearly at 0, while showing a higher range of the errors than for the other FE approaches.

Considering the effect of neglecting the CSF compartment (error of 3 layer solution with 4 layer solution as reference), all approaches show a RDM clearly below this benchmark except for some outliers and the double-layer BEM at the highest eccentricity. The ln(MAG) of the CSF effect is out of the depicted range in positive y-direction, i.e., by far higher than all presented numerical MAG errors.

To evaluate the results of the realistic head model study, we depict the cumulated relative frequency of the errors (Fig. 2). The reference was computed using the Venant approach with the high resolution FE model, the CSF effect is calculated with the Venant approach and the high resolution FE model with CSF compartment.

The results of this study are in good accordance to the results of the sphere study. Regarding the RDM, the symmetric BE approach performs best, again, having an error below 0.025 for over 95% of the dipole positions, while it shows a clear bias towards lower magnitudes. In contrast, the subtraction approach shows only slightly higher RDM errors, while achieving a low and unbiased ln(MAG) (90% within the range from -0.01 to 0.01). The same is valid for the Venant approach with, again, slightly higher overall errors than the subtraction approach, but still with 95% of the RDMs being below 0.04. Finally, the partial integration approach shows the worst results of the FE approaches.

The double layer BEM shows a good performance when looking at the RDM, being in the range of the FE approaches, while it shows strong ln(MAG) errors with a clear bias.

Most importantly, a glance at the CSF effect shows that all numerical errors lie clearly below the model error. For 95% of the dipole positions the RDM error introduced by neglecting the CSF compartment is larger than 0.05 and the caused ln(MAG) error is larger than 0.1 and strongly varying (not completely visible in graph range).

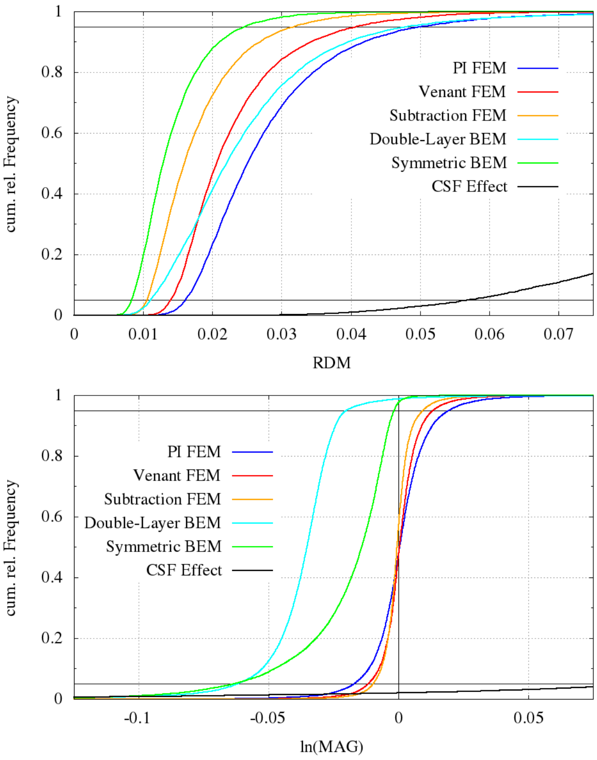

Figure 1: Histograms of RDM and ln(MAG) in realistic three layer head model.

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4 Discussion

We have shown that, with regard to RDM errors, the symmetric BEM outperforms both the double-layer BEM approach and the FE approaches in a three layer sphere model. The combination of symmetric integral representation and adaptive integration scheme leads to a high numerical stability. While the improvement compared to the double layer approach is quite big, the FE approaches show only marginally higher errors. Here, subtraction and Venant approach achieve a comparable accuracy, while the computational demand of the subtraction approach is by far higher [3]. With regard to the MAG, Venant and subtraction FE approach show a slightly smaller bias at a slightly larger variability than the symmetric BEM.

The realistic head model study shows that both the BEM and the FEM approaches perform very well in a three layer realistically shaped head model. Though the symmetric BEM is discriminated by choosing the solution of a FE approach as reference, it still outperforms the other approaches regarding RDM. Surely, these results can only be seen as hints, since no exact solution exists that can be taken as reference. Combining the results of both studies, we can conclude that both BE and FE approaches perform very well. Taking computation time into account (not depicted) the symmetric BEM and the Venant FEM offer the best performance.

Most importantly, we have demonstrated the large effect caused by neglecting the CSF, showing that a reduction of the model error will have a much higher impact than a further increase of the numerical accuracy. Since the conductivity of human CSF at body temperature is well-known to be 1.79 S/m (average over 7 subjects, ranging in age from 4.5 months to 70 years, with a standard deviation of less than 1.4% between subjects and for frequencies between 10 and 10,000Hz) and thus having nearly no inter-individual variation [19], it seems to us that the focus should be more on the reduction of the model error using, e.g., the well-performing Venant FE approach.

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